# A DISTRIBUTED BINOMIAL MODEL FOR DETERMINING SPARING LEVELS

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**Abstract**: A Distributed Binomial Model is introduced in this paper as an improvement over the simple Binomial Model in calculating the appropriate number of spares for terminal equipment. This new model takes into consideration the distributed nature of failures during the 'repair and return' interval and thus provides a more realistic estimate of the risk of not having a spare available when needed. The Distributed Binomial model has been employed as a tool in determining the level of spare equipment required.

## 1. INTRODUCTION

The Binomial distribution is often used to determine the recommended number of spares for terminal equipment. One benefit of this method is that it ties the sparing recommendation to the stated failure rate of the equipment. Another benefit is that it provides explicitly an evaluation of the risk of not having a spare available when needed as a function of both failure rate and assumed 'repair and return' (R&R) interval.

In this paper, we determine the sparing level for typical values of failure rate and R&R interval. We then introduce a Distributed Binomial model that provides a more realistic estimate of the risk an compare results.

## 2. SIMPLE BINOMIAL MODEL

In the following, residual risk (*Risk*) is defined as the probability that there will be no spare available in a station when one is required. One of the most attractive features of the Simple Binomial Model for computing the residual risk inherent is that it gives results in closed, analytical form. In addition to obviating the need for more complex numerical computations, the analytical nature of the results allows for insight into how the results depend on the inputs in a way that is not possible when relying on a purely numerical technique. For maximum utility, of course, the derivation of the analytical form should be based as closely as possible on the essential elements of the process being modeled. These include the number of operating circuit packs of a given type in the cable station, the failure rate of the circuit pack, and the realistic interval to repair and return a failed pack.

In the Simple Binomial model, the residual risk is computed as the probability that the number of failures of a specific pack occurring in a given station will exceed the number of spares of that specific pack assigned to that station. Mathematically, for a station where there are $n$ operating packs and $M$ spares, this residual risk reduces to:

$$Risk_{SBi} = 1 - \sum_{m=0}^{M} \binom{n}{m} \hat{p}^m (1-\hat{p})^{n-m}, \quad (1)$$

where $\binom{n}{m}$ are the well-known Binomial coefficients.

$$\hat{p} = \lambda * t_{R\&R} \qquad (2)$$

where $\hat{p}$ is the probability of failure of an individual pack over the R&R interval based on its failure rate, λ [1]. From Eq. (2), it is clear that changes by the same factor in the estimated failure rate or the R&R interval will have identical effect on the residual risk. Trade-offs in these input parameters can be made to optimize the risk. The dependence of the recommended number of spares on $\hat{p}$ is essentially linear until the relationship bottoms at one spare.

Operationally, the *Risk* is calculated for increasing values of *M*, until the *Risk* falls below a specified tolerable level. For example, *Risk* might be determined by allowing the situation of having no spares when one is needed to occur once in a 25-year system life. This requirement translates to a 2% residual risk for a 6-month R&R interval.

Implicit in this Simple Binomial model is the assumption that the full complement of spares of a given type assigned to a specific station is available for use on the first day of each R&R interval and that it is only the total number of failed packs during that interval and not the exact timing of the failures which determines the residual risk. In its simplest form, therefore, the Simple Binomial model for computing residual risk is open to the criticism that it does not capture one of the elements of the R&R process — namely the distributed nature of failures over the R&R interval. Thus, using this assumption might under estimate the residual risk. The potential under estimation results because failures which do not occur at the beginning a given R&R cycle will reduce the number of spares of the affected pack available during the following R&R interval.

## 3. DISTRIBUTED BINOMIAL MODEL

The Distributed Binomial model refines the Simple Binomial model to include the effect of having failures distributed throughout the R&R interval, so that the number of spares available at any time during the interval depends on the number of failures observed in the preceding interval. In this model, packs that failed during the preceding interval are treated mathematically as unavailable for use until the appropriate return time in the current R&R interval. Thus, a pack that failed at the half-way point of the previous R&R interval would not be available for use until at least the half-way point of the current interval.

As a simple example of how the Distributed Binomial model could be implemented, consider the case where a total of 2 spare packs of a specific circuit at a given station are provided to support the total number of operating packs of that type at that station. Then:

- If the preceding R&R interval experienced zero (0) failures of this type, 2 spare packs are available at the beginning of the current R&R period and 2 failures could occur without adding to residual risk

- If the preceding R&R interval experienced one (1) failure of this type, we assign to it a failure time equal to the half-way point of the previous R&R interval. In this case, there is only 1 spare available at the beginning of the current R&R interval and only 1 failure could be tolerated during the first half of the interval. Up to 2 failures are allowable during the second half of the current R&R interval.

- If the preceding R&R interval experienced two (2) failures of this type, we assign the failure times to be at the one-third point and two-thirds points of the previous interval. In this case, there are no spares available at the beginning of the current R&R interval. Zero (0) failures are allowable during the first one-third, 1 failure is allowable between the one-third and the two-thirds points, and 2 failures are allowable during the last one-third of the interval.

In this way, the computation of the residual risk involves both sums and products of the probabilities of observing various numbers of failures during various time intervals. Figure-1 shows a schematic for 3 failures within one R&R interval, it can be seen that the extension of the method outlined above in the case of 2 spares to larger numbers of spares is straightforward.

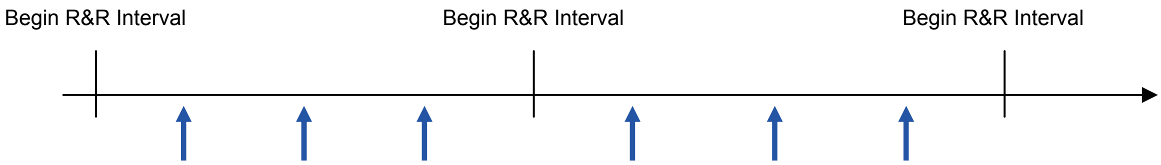


**Figure 1 A schematic for the Distributed Binomial Model for 3 failures**

Unlike the simple binomial model, the distributed binomial model accounts for the fact that failures can occur at any point during the R&R interval and thus that all spare cards may not be available at the beginning of each R&R interval.

## 4. RESULTS AND DISCUSSION

As an example, recommended spares and the associated residual risk for 3- and 6-month R&R intervals are calculated for representative failure rates and for different numbers of active packs. The number of spares and their associated residual risk are calculated for both Simple Binomial and Distributed Binominal models and are listed in Table-1 and Table-2 below.

**Table 1 Calculated Number of Spares Required and Residual Risk for a 3-Month R&R Interval**

| Active Units | Failure Rate (FITs) | Simple Binomial | | Distributed Binomial | |
|---|---|---|---|---|---|
| | | # Spares | Residual Risk | # Spares | Residual Risk |
| 8 | 10,000 | 1 | 1.2% | 2 | 0.2% |
| 24 | 10,000 | 2 | 1.5% | 3 | 0.6% |
| 48 | 10,000 | 4 | 0.4% | 5 | 0.2% |

**Table 2 Calculated Number of Spares Required and Residual Risk for a 6-Month R&R Interval**

| Active Units | Failure Rate (FITs) | Simple Binomial | | Distributed Binomial | |
|---|---|---|---|---|---|
| | | # Spares | Residual Risk | # Spares | Residual Risk |
| 8 | 10,000 | 2 | 0.1% | 2 | 0.6% |
| 24 | 10,000 | 3 | 1.3% | 4 | 0.7% |
| 48 | 10,000 | 5 | 1.6% | 6 | 1.2% |

The imposed requirement for the calculation is that the residual risk for an R&R interval must be less than 2%.

## 5. CONCLUSIONS

As expected, the Distributed Binomial model increases the number of spares needed to stay below a fixed residual risk (such as 2% in the example) over that computed using the Simple Binomial approach. This increase reflects the more realistic assumption in that the failures could happen any time within a R&R interval and that the availability of the spares is not automatically guarantied at any given time within a R&R interval.

## 6. REFERENCES

[1] B. Dean, C. Breverman, "Reliability Performance of Tyco Telecommunications' Generation 3 DPSK HPOE", SubOptic 2010, Yokohama, Japan.